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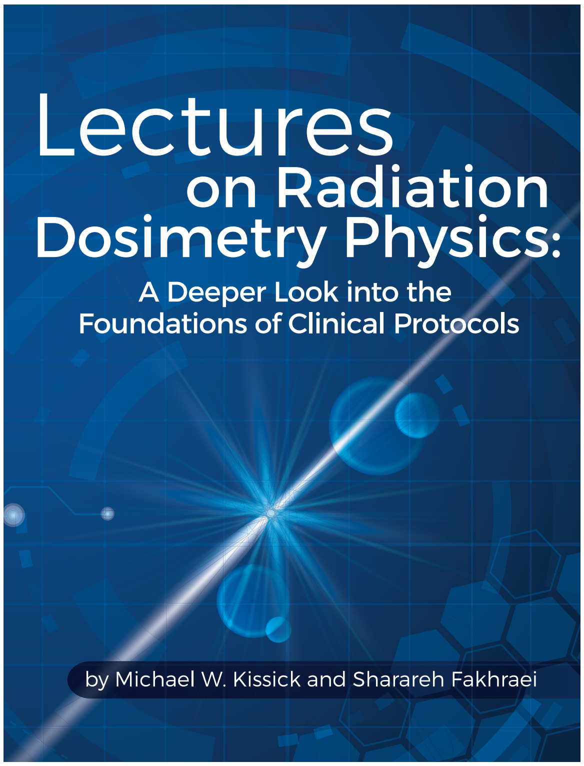

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8

Charged Particle Interactions with Matter

The photon interactions transfer energy to electrons, and it is these electrons that deliver dose as they slow down in matter. To understand the basic physics involved, we will start with an important derivation for heavy charged particles that will then be generalized to eventually work for electrons slowing down in condensed matter. It is in fact remarkable that this extension to electrons works as well as it does.

Interactions of Heavy Charged Particles

In Figure 8.1, let's consider a heavy charged particle with kinetic energy, *T*, and velocity, *v*, going very quickly by an atom with an impact parameter, *b*. Here we use simple description of the nucleus, which is the "liquid drop model." From this model, we get the nuclear radius is $R_{nuc} \approx 1.4 \times 10^{-13} A^{1/3}\text{cm}$ where A is the mass number†. The radius of the atom is about $a \approx 0.18\beta\lambda$ from Bohr's theory, where λ is the wavelength of the electron in an outer shell and $\beta = v/c$ is the particle speed compared to the speed of light. The radius of the atom is much larger than the nuclear radius. It is like a baseball (nucleus) in Camp Randall stadium (atom). Note that there is a velocity dependence to all of this, and this is just order of magnitude validity anyway.

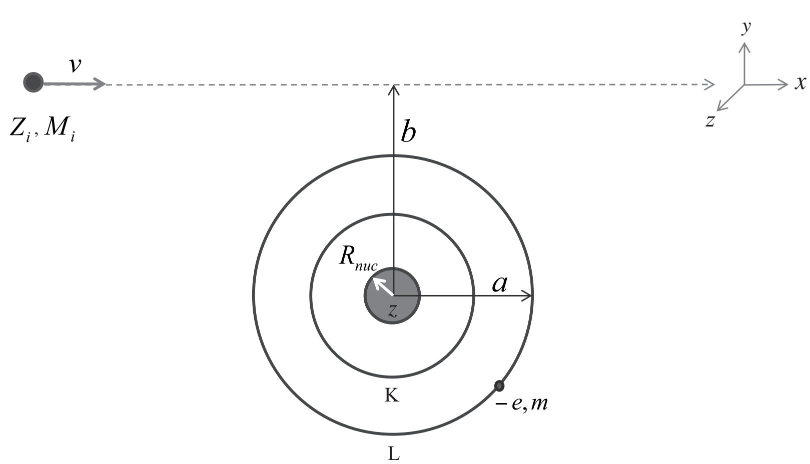

Figure 8.1. Geometry of a heavy charged particle interacting with a single atom. Note: this figure is not to scale.

†Mass number variable, A, here is the number of protons plus the number of neutrons, and it is an integer for just this discussion.

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The "heavy and fast" particle at the base of the arrow in Figure 8.1 has a charge, Z_i, and a mass, M_i, and a significant energy, $\gamma M_i c^2$, such that $\gamma = 1/\sqrt{1 - v^2/c^2} >> 1$. Notice that "fast" here means Born approximation†. The atomic bound electron, on the other hand, has only a charge, $-e$, a mass, m, and bound energy such that kinetic energy, T, roughly equals its potential energy. In other words: $mv^2/2 \sim e^2/a_0$ where $a_0 \approx \hbar^2/me^2$, which gives $v^2/c^2 \sim e^4/\hbar^2 c^2 \sim (1/137)^2$, the fine structure constant, α squared. Therefore, the incident particle needs to have more kinetic energy than this ($T > mc^2\alpha^2$). This implies that only about 53 keV is needed for the incident particle in this treatment. Other generalizations beyond these assumptions will also be possible‡.

In general we will be categorizing reactions according to impact parameter§ b:

1. **Soft collisions ($b >> a$)**: Continuous energy loss with many atoms at once—interactions with orbiting electrons. Soft collisions lead to excitations. Soft collisions are about half of what is happening.
2. **Hard collisions ($b \approx a$)**: Hard collisions are also near half of the interactions, but large events that can lead to the liberation of other charges with their own trajectories are more rare. We will need to carefully account for this. The liberated secondary charges are outer-shell-orbiting electrons. Hard collisions give ionizations with resulting excitations.
3. **Nuclear electric field interactions ($b \approx R_{nuc}$)**: This results in *elastic* collisions (mostly) and bremsstrahlung (only 2–3% for radiation therapy energies). Note that only light particles, like electrons, will show any significant bremsstrahlung.
4. **Nuclear interactions ($b < R_{nuc}$)**: Nuclear reactions will only occur here.

Stopping Power and Mass Stopping Power

One of the most important concepts in the physics of dosimetry is **stopping power**, $\frac{dT}{dx}$. Stopping power is defined as the rate of energy loss for distance traveled into the medium. The mass stopping power, $\frac{dT}{\rho dx}$, is then obtained by dividing stopping power by the density of the material. Mass stopping power is usually divided into two terms: one for (soft and hard) collisional energy losses and the other for radiative (bremsstrahlung) energy losses as follows:

$$\frac{dT}{\rho dx} = \left(\frac{dT}{\rho dx}\right)_c + \left(\frac{dT}{\rho dx}\right)_r . \tag{8.1}$$

The units of stopping power and mass stopping power are MeV/cm and MeV/(g/cm^2), respectively.

Radiative stopping power is the production of bremsstrahlung. Heavy charged particles do not produce much bremsstrahlung: i.e., a proton is ~2000 times heavier than an electron and, therefore, 4×10^6 times less bremsstrahlung is produced. Thus, if M is much heavier than an electron, then we will assume radiative loss is negligible:

$$\left(\frac{dT}{\rho dx}\right)_r \sim \frac{1}{(Mc^2)^2} \approx 0. \tag{8.2}$$

However, for electrons and positrons, this radiative term will not be negligible. Later in this chapter, we will discuss the mass stopping power for electrons and positrons.

†For more discussion about the Born approximation, visit Evans (1955), page 887.
‡Note we are using slightly different notations: T and m and z versus Z.
§Note that this categorization is general and not just for heavy charged particles.

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Collisional Mass Stopping Power for Heavy Charged Particles

Returning to Figure 8.1, consider the momentum impulse as we integrate in time the rate of momentum change (Gaussian units here):

$$\vec{F} = \frac{d\vec{P}}{dt} = -e\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right). \tag{8.3}$$

However, only the electric field part is important, since $v/c \sim \alpha$. For the geometry given above, and for a given b, the kicks in x and y are given by the following:

$$E_x = \frac{-Ze\gamma vt}{\left(b^2 + \gamma^2 v^2 t^2\right)^{3/2}}. \tag{8.4}$$

$$E_y = \frac{-Ze\gamma b}{\left(b^2 + \gamma^2 v^2 t^2\right)^{3/2}}. \tag{8.5}$$

One then integrates from – infinity to + infinity in time while noticing that the kick in E_x vanishes because it is an odd function. Therefore, only the y-kick comes into the momentum change:

$$\begin{aligned} \Delta P = \Delta P_y &= \int_{-\infty}^{\infty} dt\ eE_y(t) \\ &= 2\frac{Ze^2 b}{v}\int_0^{\infty} \frac{d(\gamma vt)}{\left(b^2 + (\gamma vt)^2\right)^{3/2}} \\ &= \frac{2Ze^2}{bv}. \end{aligned} \tag{8.6}$$

This is like an impulse with $P_{y\_initial} = 0$. The energy lost is simply obtained then by $(\Delta P)^2$ / 2m to arrive at the following very important result for the interaction with a single electron:

$$\Delta T(b) = \frac{(\Delta P)^2}{2m} = \frac{2Z^2 e^4}{\mathrm{mv}^2}\left(\frac{1}{b^2}\right). \tag{8.7}$$

Notice that this energy loss is independent of the incident particle mass, M_i, even though a large mass is assumed, given that the incident particle does not change directions in this approximation. Also notice that the energy loss is proportional to $1/v^2$. This is a very important result that remains dominant as we proceed further. However, that was only for a given b, but in our problem, we have a whole range of b values. Therefore, we need to integrate over the full range of the impact parameter, b.

According to Figure 8.2, the number of electrons in the differential annulus of volume $(dx) \cdot (2\pi bdb)$ is $[(2\pi bdb) \cdot (dx)] \cdot \rho \cdot (N_A z/A)$. The differential energy loss experienced by the particle having an impulse with this annulus is then[†] the following:

$$dT = \left[(2\pi bdb)\cdot(dx)\right]\cdot\rho\cdot\left(N_A z/A\right)\cdot\frac{2Z^2 e^4}{mv^2}\left(\frac{1}{b^2}\right). \tag{8.8}$$

[†]Note that the small Z is the material atomic number here, opposite from most books.

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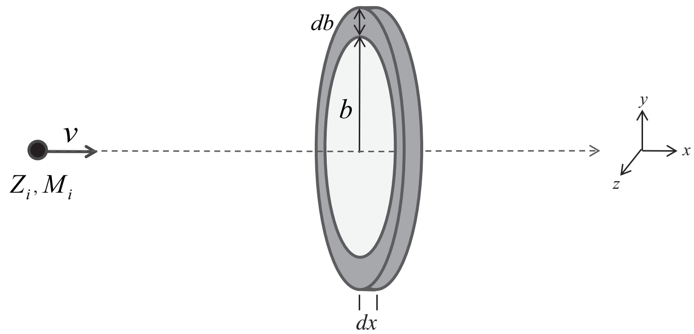

Figure 8.2. Geometry of a heavy charged particle interacting with a differential ring of atoms.

Rearranging and noting that we need to integrate over all possible b values:

$$\left(\frac{dT}{\rho dx}\right)_c = 4\pi\left(\frac{N_A z}{A}\right)\frac{Z^2 e^4}{mv^2}\int_{b=0}^{\infty}(db/b). \tag{8.9}$$

The critical issue is how to perform the integral. Since with integrating from zero to infinity, the stopping power would diverge. Instead of the full integral, we will then take the integral from a b_{min} to b_{max}, which results in the following:

$$\left(\frac{dT}{\rho dx}\right)_c = 4\pi\left(\frac{N_A z}{A}\right)\frac{Z^2 e^4}{mv^2}\ln\left(\frac{b_{\max}}{b_{\min}}\right). \tag{8.10}$$

According to the Equation (8.8), if b goes to zero, then dT will rise without any limit. However, there is a certain maximum for energy loss and that occurs with a head-on collision. So we need to choose a minimum for b which gives the maximum magnitude for energy loss.

After derivations of energy transfers with bound electron shell states in quantum mechanics, and some complicated Bessel function math†, the stopping power is given by the following:

$$\left(\frac{dT}{dx}\right) = 4\pi(n)\frac{Z^2 z e^4}{mv^2}\left[\ln(B_c) - \beta^2/2\right], \tag{8.11}$$

where,

$$B_c \equiv \frac{b_{\max}}{b_{\min}} = (1.123)\frac{\gamma^2 m v^3}{Z_i e^2 \langle\omega\rangle}. \tag{8.12}$$

Here, $\langle\omega\rangle$ is the average motion frequency of electrons, and n is the number density of atoms‡. Note and remember from earlier chapters that $(\gamma - 1) = T/Mc^2$ and $\gamma = 1/\sqrt{1-\beta^2}$.

†For more discussion regarding this topic, see J.D. Jackson's *Classical Electrodynamics*, Chapter 13.
‡The factor 1.123 is from 1.123 = 2/exp(0.577...), and 0.577... is Euler's constant.

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Table 8.1: Maximum Energy Transferred for Various Situations*

Particle	Value of T'_{max}
For electrons	$T'_{\max} = T/2$
For positrons	$T'_{\max} = T$
For a heavy charged particle with mass M	$T'_{\max} = T\left(\frac{1+(2Mc^2/T)}{1+(M+m)^2c^2/(2m/T)}\right)$
For a heavy charged particle with mass M, which $M >> m$ and $T << Mc^2$	$T'_{\max} \approx 2mc^2\frac{\beta^2}{1-\beta^2} \approx 2mv^2$

*Note that for positrons, the maximum energy transferred to an electron is twice what it is for electron to another electron. That is because the particles are different, and the secondary is not simply the one with the lower energy.

Let's separate the collisional mass stopping power into two terms: one for hard collisions and one for soft collisions. We will start out with an arbitrary energy cutoff, H, that will separate the two by an impact parameter or, equivalently in this case, an energy. If σ_s and σ_h are cross sections for soft and hard collisions, respectively:

$$\left(\frac{dT}{\rho dx}\right)_c = \left(\frac{dT_s}{\rho dx}\right)_c + \left(\frac{dT_h}{\rho dx}\right)_c = \left(\frac{N_A z}{A}\right)\left[\int_{T'_{\min}}^{H} \frac{d\sigma_s}{dT'} T'dT' + \int_{H}^{T'_{\max}} \frac{d\sigma_h}{dT'} T'dT'\right]. \tag{8.13}$$

The quantity T' here is the energy transferred by the fast charged particle to electrons. The quantity H here is somewhat arbitrary, but it should be about the value of the minimum energy at which the electron is ejected for hard collisions. Table 8.1 illustrates the values of T'_{max}.

Soft Collision Mass Stopping Power for Heavy Charged Particles

If the energy transferred, T', is less than the ionization potential, only excitations can occur. If it is larger, then ionizations can occur, but they will also have excitations as well, and the energy must be larger than H.

The quantity I is defined as the **mean excitation potential** of the absorbing medium; it includes both excitations and ionizations. See Attix (2004), Appendix B.1 and B.2. There is a lot of uncertainty to I, but at least it is in the logarithm, which reduces the sensitivity to that uncertainty. I is approximately proportional to z, the atomic number of the medium.

Derived using the Born approximation† ($\beta >> zZ/137$)), then the soft collision mass stopping power is the following:

$$\begin{aligned}\left(\frac{dT_s}{\rho dx}\right)_c &= 2\pi r_0^2 mc^2\left(\frac{N_A z}{A}\right)\frac{Z^2}{\beta^2}\left[\ln\left(\frac{2mc^2\beta^2}{I^2(1-\beta^2)}H\right)-\beta^2\right] \\ &= (0.1535)\left(\underbrace{\frac{z}{A}}_{material}\cdot\underbrace{\frac{Z^2}{\beta^2}}_{particle}\right)\left[\ln\left(\frac{2mc^2\beta^2}{I^2(1-\beta^2)}H\right)-\beta^2\right].\end{aligned} \tag{8.14}$$

†The particle's kinetic energy is much greater than an electron's potential (orbital) energy.

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Here the quantity $2\pi r_0^2 mc^2 N_A = 0.1535$ (MeV/(g/cm^2)). For ease later, we can define the following:

$$k = (0.1535)\left(\frac{z}{A}\cdot\frac{Z^2}{\beta^2}\right). \tag{8.15}$$

The units of k are MeV/(g/cm^2). In fact, very important dependencies are in k. The $\frac{z}{A}$ term is a property of only the material, and $\frac{Z^2}{\beta^2}$ is a property of only the incident particle, and it is this part that produces the Bragg peak. The above applies to electrons, positrons, and heavy charged particles. Soft collisions are most, or at least half, of the collisions for radiation therapy energies.

Hard Collision Mass Stopping Power for Heavy Charged Particles

A hard collision is often further defined as when an electron is ejected with a considerable fraction of the maximum energy transferable, $T_{\max}$. These recoil electrons are called "delta-rays" or "knock-on" electrons, and they can be seen in bubble chambers.

The **differential hard collision cross section per electron** for heavy charged particles is given by the following:

$$\frac{d\sigma_h}{dT'} = 2\pi r_0^2 mc^2 \frac{Z^2}{\beta^2}\left(\frac{1-\beta^2(T'/T'_{\max})}{(T')^2}\right). \tag{8.16}$$

There is some dependence on particle spin for this differential hard collision cross section per electron. We will use the form for zero spin particles (like alpha, pion, …), but it will apply to spin ½ particles (like protons, electrons, muon, …) provided $T' << Mc^2$.

Then the **hard collision mass stopping power** for $H << T'_{\max}$ is the following:

$$\begin{aligned}\left(\frac{dT_h}{\rho dx}\right)_c &= \left(\frac{N_A z}{A}\right)\left[\int_H^{T'_{\max}} \frac{d\sigma_h}{dT'} T'dT'\right] \\ &= k\left[\int_H^{T'_{\max}} dT'/T' - (\beta^2/T'_{\max})\int_H^{T'_{\max}} dT'\right] \\ &= k\left[\ln(T'_{\max}/H) - (\beta^2/T'_{\max})(T'_{\max}-H)\right] \\ &= k\left[\ln(T'_{\max}/H) - \beta^2\right].\end{aligned} \tag{8.17}$$

Now with combining Equations (8.13), (8.14), and (8.17), mass stopping power for heavy charged particles can be written as the following:

$$\left(\frac{dT}{\rho dx}\right)_c = \left(\frac{dT_s}{\rho dx}\right)_c + \left(\frac{dT_h}{\rho dx}\right)_c = k\left[\ln\left(\frac{2mc^2\beta^2}{I^2(1-\beta^2)}H\right) - \beta^2\right] + k\left[\ln(T'_{\max}/H) - \beta^2\right]. \tag{8.18}$$

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Table 8.2: Magnitudes for Parameters Related to High-energy Particle Speed Relative to the Speed of Light

β	T_p	β^2	$\ln(\beta^2/(1-\beta^2))$	$\ln(\beta^2/(1-\beta^2))-\beta^2$
0.80	626	0.64	0.57	–0.07
0.90	1214	0.810	1.45	1.17
0.95	2067	0.903	2.23	1.33
0.99	5713	0.980	3.89	2.91

With a bit of math, and recalling that for a heavy charged particle $T'_{\max} \approx 2mc^2 \dfrac{\beta^2}{1-\beta^2} \approx 2mv^2$, the Equation (8.18) simplifies as follows:

$$\left(\frac{dT}{\rho dx}\right)_c = 2k\left[\ln\left(\frac{2mc^2\beta^2}{I(1-\beta^2)}\right)-\beta^2\right]. \tag{8.19}$$

There is also a more handy form for mass collisional stopping power for spin ½ and $T' << Mc^2$ as follows:

$$\left(\frac{dT}{\rho dx}\right)_c = (0.3071)\underbrace{\left(\frac{z}{A}\right)}_{\text{medium}}\underbrace{\left(\frac{Z^2}{\beta^2}\right)}_{\textit{particle}}\left[13.84+\ln\left(\frac{\beta^2}{1-\beta^2}\right)-\beta^2-\ln I\right]. \tag{8.20}$$

Here, mean excitation potential, I, has units of eV. Note that the energy dependence in the brackets of Equation (8.20) is weak (see Table 8.2).

Shell Correction

The complex motion of the orbital electrons is accounted for with the shell correction. When the particle velocity is less than the orbital velocity of the electrons in that shell, then those electrons do not participate significantly in collisions with the particle.

The shell correction factor is written as C/z, and it (mostly) decreases the mass collisional stopping power by a small amount as follows:

$$\left(\frac{dT}{\rho dx}\right)_c = 2k\left[\ln\left(\frac{2mc^2\beta^2}{I(1-\beta^2)}\right)-\beta^2-C/z\right]. \tag{8.21}$$

The shell correction is a function of the particle velocity and the atomic number of the medium. See Attix (2004), page 172, Figure 8.3 for the magnitude and proton kinetic energy dependence of the shell correction, as an example. Note that it is more important at lower energies.

Dependence of the Stopping Power on the Medium

The factor z/A has a value near 0.5 ±0.05 for most elements, dropping to lower values at higher z. Hydrogen-1 has the highest value of 1, which is why it is used to slow or shield fast charged particles. See appendix B.1 in Attix (2004), page 527, for z/A values.

The term ln$I$ also makes high-z materials have lower stopping power. Also, the shell correction generally decreases the stopping power.

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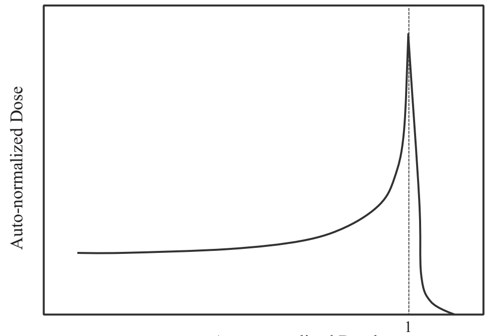

Figure 8.3. Dose depth curves for heavy ions along the centered axis of a broad beam. Note that the Bragg peak is due to the $1/\beta^2$ in *k*.

Mass Collisional Stopping Power Dependence on Particle Velocity

The term $1/\beta^2$ is dominant at low energies. It causes the mass collisional stopping power to sharply increase as the particle slows down. The result is the so-called "Bragg Peak" (see Figure 8.3).

When $\beta = v/c \approx 1$, the energy increases quickly as the speed creeps up against *c*. The linac accelerating microwave cavity uses this fact to keep the particle in phase with the phase velocity of the standing or traveling microwave. The cavity is loaded with periodic barriers to slow the phase velocity below *c*.

Also, when $\beta \approx 1$, the $1/\beta^2$ term has little influence, but the $\beta^2 / (1-\beta^2)$ term increases. See Table 8.2.

The kinetic energy of a particle is directly proportional to its rest mass.

There is no mass dependence on the heavy charged particle stopping power. Therefore, any heavy particle with the same velocity and charge will have the same stopping power, *but the scatter and range straggling could be different.*

Mass Collisional Stopping Power Dependence on Particle Charge

At low energies (speeds $\beta < 0.1$), there is an effective charge, Z^*, to be used instead of the charge, *Z*, because of the attachment of the incident particle's electrons. Higher-energy particles are more likely to be fully ionized. See Anderson (1984), page 21, Figure 2.4.

The Z^2 factor means that particles with multiple charges have a much higher mass collisional stopping power than singly charged particles. For example, if $\beta_{\alpha^{2+}} = \beta_{p^+}$, then:

$$\left(\frac{dT}{\rho dx}\right)_c^{\alpha^{2+}} = 4 \cdot \left(\frac{dT}{\rho dx}\right)_c^{p^+} . \tag{8.22}$$

This fact can be used to obtain stopping powers for any heavy charged particle from a table of mass collisional stopping powers for protons. For this purpose, do the following steps:

1. Look up or calculate β for particle *x* with kinetic energy, T_x.
2. Look up or calculate the proton kinetic energy, T_p, for the same β.
3. Look up the mass stopping power for a proton with kinetic energy T_p.
4. Multiply the mass stopping power for a proton by $(Z_x^* / Z_p^*)^2$. Note that Z_p^* is unity, and Z_x^* is the effective charge on particle *x* at its speed.

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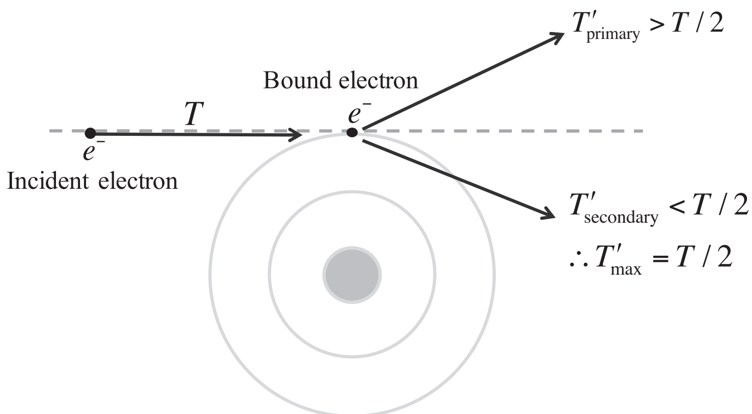

Figure 8.4. Kinetic energy after a hard collision by an electron. By convention, the primary charge has more energy than the secondary charge.

It should be emphasized that through the decades, the stopping power formula has been progressively refined. One of the most important refinements was to cure the high-speed (v approaching c) limit. As explained in Jackson (1999), page 636, the very fast particle causes so much ionization from relativistic effects that it self-shields its charge from the plasma effect of the ionizations it creates. The time scale associated with this shielding is manifest by the plasma frequency quantity that appears in this limit. Yet, the fundamental relations covering most of the important physics for most applications and for most particles are described by the simple derivation at the start of chapter, and this fact is remarkable in the history of modern physics.

Electron and Positron Interactions

In hard collisions by *electrons*, one cannot tell which was the primary or which was the secondary—by convention, the one with the highest energy (that is, the faster electron) is the "primary." Therefore, the maximum kinetic energy transferable is the amount that the lower-energy electron can have (Figure 8.4).

However, in the *positron* case, we *do* know which particle is which, and we cannot reassign the primary label, so the maximum kinetic energy transferred is $T'_{\max} = T$ (look at Table 8.1). Therefore, the maximum kinetic energy transferred is $T/2$ for electrons, and it is T for positrons.

The differential cross section for the electron hard collisions—which describes the collision between two free electrons—was derived by Möller (1932) as follows:

$$\frac{d\sigma_h}{dT'} = 2\pi r_0^2 mc^2 \frac{1}{\beta^2 (T')^2}\left(\frac{T}{T-T'}\right)^2 \left\{1-\left[3-\left(\frac{T}{T-mc^2}\right)^2\right]\frac{T'}{T}\left(1-\frac{T'}{T}\right)+\left(\frac{T}{T-mc^2}\right)^2\left(\frac{T}{T-T'}\right)^2\right\}. \tag{8.23}$$

Note that z dependence here is implicit in β^2 via mass stopping power. The differential cross section for positron-electron collisions is even more complex, and it was derived by Bhabha (1936).

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